HEAT TRANSFER AND FRICTION IN THE TURBULENT BOUNDARY LAYER OF A COMPRESSIBLE GAS AT A PERMEABLE SURFACE

B. P. Mironov and N. I. Yarigina

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Test data on heat transfer and friction are generalized for a wide range of the injection factor with the Mach number equal to 0.3, 2.05, 3.05, and 4. The test results agree with the theoretical analysis also given here.

The problem considered here is of great practical importance. Boundary layers of this kind are formed during porous cooling, during the burnout of heat resistant coatings on objects returning from outer space, etc. Although numerous studies have been made concerning this problem (see [1]), many of its aspects have not yet been clearly explained. It is not the quantitative discrepancy between various existing data which concerns us here. That can be explained by the peculiarities of the particular test conditions. Much more significant seem here the following fundamental questions: how does injection affect the recovery factor, does the modified Reynolds analogy apply here, is it sufficient to account for the effect of anisothermality on friction and heat transfer by simply assigning standard values to the St_0 number and the C_{f_0} factor on the basis of the relations derived in [2, 3] with the results in [4] taken into consideration, etc?

We will present here the results of an experimental study concerning the friction and the heat transfer in a supersonic turbulent boundary layer at a porous surface, with the Mach number equal to 0.3, 2.05, 3.05, or 4 and with the injection parameter (air into air) varying over a wide range of values. A theoretical analysis of these processes will be made by applying the Kutateladze–Leont'ev theory to a turbulent boundary layer of a fluid with a vanishingly low viscosity [1].

For a supersonic anisothermal turbulent boundary layer with a transverse fluid flow, assuming that similarity exists between the velocity field and the enthalpy field, we have

$$\int_0^1 \frac{d\omega}{\sqrt{(\Psi + b\omega)\,[\psi + (\psi - \psi^*)\,\omega - (\psi^* - 1)\,\omega^2]}} = 1. \tag{1}$$

by analogy with [2, 3] ($Re \to \infty$). Here $\psi^* = 1 + r(k-1/2)M^2$. From Eq. (1) follows for the referred heat transfer (friction) function $\Psi = (St/St_0)_{Re^{**}}$

$$\sqrt{\Psi} = \frac{1}{\sqrt{b_1(\psi^* - 1)}} \cdot \frac{2}{\sqrt{|\omega_2| + |\omega_1|}}\,[F(\varphi_1;\ P) - F(\varphi_2;\ P)]; \tag{2}$$

where F is an incomplete elliptic integral of the first kind,

$$\varphi_1 = \arcsin\sqrt{\frac{|\omega_2|}{|\omega_2| + 1/b_1}}; \qquad \varphi_2 = \arcsin\sqrt{\frac{|\omega_2| - 1}{|\omega_2| + 1/b_1}};$$

$$P = \sqrt{\frac{|\omega_2| + 1/b_1}{|\omega_2| + |\omega_1|}};$$

and ω_2, ω_1 are roots of the equation $\rho_0/\rho = 0 = f(\psi, \psi^*, \omega)$.

Institute of Thermophysics, Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk.
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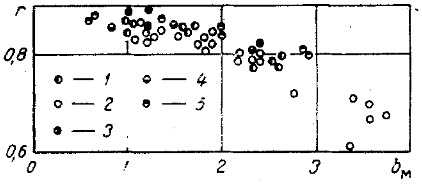

Fig. 1

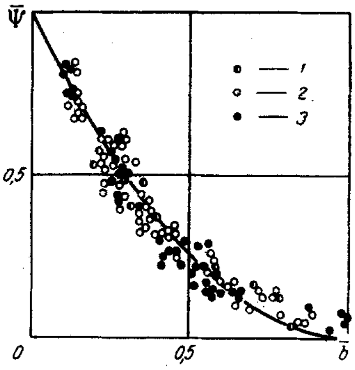

Fig. 2

Fig. 1. Recovery factor as a function of the injection parameter: Ma = 2.05 (1), 3.05 (2), 4.0 (3). Data according to [7]: Ma = 2.0 (4), 3.2 (5).

Fig. 2. Heat transfer as a function of injection: Ma = 2.05 (1), 3.05 (2), 4.0 (3), and $\bar{\psi}$ = 0.9-1.85. Solid line represents calculation according to Eq. (3).

It is interesting to note that the exact solutions for function Ψ according to (2) can be approximated by the simple relation

$$\Psi = \Psi_{\bar{\psi}} \Psi_{M} \Psi_{B}, \qquad (3)$$

which represents a combination of already well-known referred friction and heat transfer functions [2-6]:

$$\Psi_{M} = \left[\frac{\operatorname{arctg}\left(M \sqrt{r \frac{k-1}{2}} \right)}{M \sqrt{r \frac{k-1}{2}}} \right]^2;$$

$$\Psi_{\bar{\psi}} = \left(\frac{2}{\sqrt{\bar{\psi}} + 1} \right)^2 \text{ is the Kutateladze formula;}$$

$$\Psi_{B} = \left(1 - \frac{b}{b_{cr}} \right)^2 \text{ is the Kutateladze—Leont'ev formula.}$$

The critical value of the injection parameter b_{cr} can be found from (1) with $\Psi = 0$. According to [1],

$$\sqrt{b_{cr}} = \frac{1}{\sqrt{\psi^* - 1}} \cdot \frac{2}{\sqrt{|\omega_2| + |\omega_1|}} F(\beta, n);$$

$$\beta = \arcsin \sqrt{\frac{|\omega_2| + |\omega_1|}{|\omega_2| + (|\omega_1| + 1)}}; \qquad n = \sqrt{\frac{|\omega_2|}{|\omega_2| + |\omega_1|}}. \qquad (4)$$

The exact solution for b_{cr} can, in turn, also be replaced by a simple approximation

$$b_{cr} = b_{cr_0} \Psi_{M}. \qquad (5)$$

The values of b_{cr} corresponding to critical injection at subsonic velocities are given by well-known formulas in [2].

For calculations according to formulas (3)-(5) at a given value of the Mach number, one must know the recovery factor as a function of the injection parameter r = f(b). This relation has been determined experimentally.

The tests for this study were performed on a porous cylinder 40 mm in diameter and 336 mm long, oriented parallel to the stream inside the aerodynamic tunnel at the Institute. Essential information about the instrumentation can be found in [1]. The recovery factor was measured at a constant stagnation temperature in the mainstream (100°C) and at a constant injection factor b_M, the temperature of the injected gas was varied so as to produce test modes with different directions of heat flow. From the thermal flux versus temperature curve $q = f(T_W)$ and its intersection with the axis of abscissas, we then found the

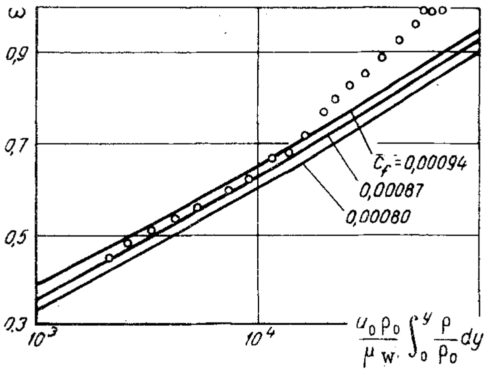

Fig. 3

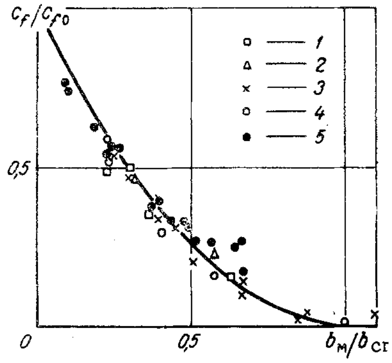

Fig. 4

Fig. 3. Typical velocity profile plotted for determining the friction coefficient (Ma = 3.05, $\bar{j} = 0.168 \cdot 10^{-2}$).

Fig. 4. Friction as a function of the injection parameter $\bar{b}$: Ma = 0.3 (1), 2.05 (2), 3.05 (3), 4.0 (4), 3.2 [10] (5). Solid line represents calculation according to Eq. (3).

stagnation temperature at the wall T_W^* and from there we determined the recovery factor. This procedure was facilitated by the almost linear relation between thermal flux and wall temperature near the axis of abscissas. The results are shown in Fig. 1 and agree with the corresponding data in [7].

With the aid of the experimentally obtained relation $r = f(b_M)$, the heat transfer data were now evaluated in the coordinates of the theoretical formulas (3) and (5):

$$\frac{\mathrm{St}}{\Psi_{\bar{\psi}}\Psi_{M}\mathrm{St}_0} = f(\bar{b}). \tag{6}$$

According to Fig. 2, the data evaluated according to formula (6) do not depend on the Mach number and confirm the validity of calculations based on formulas (3)-(5). One must emphasize that it is quite impossible to generalize the test data without taking into account relations $r = f(b_M)$ and $b_{cr} = f(\psi)$. Without these relations, a generalization is possible only for some definite test conditions which, in the final analysis, represent a certain special case. This is well illustrated in [1].

The test data on friction have been generalized analogously. The friction coefficients were determined by a transformation analogous to the Cowles transformation [8], from the velocity profiles measured under adiabatic test conditions. D. Cowles had analyzed the effect of compressibility on the magnitude of turbulent shear stress at an impermeable wall in an adiabatic flow under constant pressure, by transforming the transverse coordinate in the same manner as Dorodnitsyn had done earlier but considering the flow function not to be invariant with respect to this transformation. It has been shown in [9] that the Cowles method is valid in the zone where the "wall law" applies and yields the skin friction, if the velocity profile in the turbulent main-stream without extrapolation to the laminar sublayer is known. The basic difficulty with the Cowles method is in determining the quantity $\bar{\mu}/\sigma\mu_W$, which must be evaluated empirically ($\sigma(x) = (\bar{\psi} - \bar{\psi}_W)/(\psi - \psi_W)$ is the mapping function, μ_W is the gas viscosity at the wall, and $\bar{\mu}$ is the gas viscosity in the "incompressible transform system" (the dash above a symbol refers the given quantity to the transform system).

This dimensionless parameter is usually determined on the basis of some hypothesis as, for example, the sublayer hypothesis. It is assumed in the latter case that the Reynolds number remains constant at the edge of the laminar sublayer. In more complicated situations as when mass transfer also occurs, for example, the application of the sublayer hypothesis becomes problematic on account of insufficient available information about the behavior of laminar sublayers during the flow of compressible and incompressible fluids. It will be shown here that C_f can be successfully determined at a low Mach number (Ma = 0–4.0) by means of just one Dorodnitsyn transformation of the transverse coordinate, provided

$$\bar{\mu}/\sigma\mu_W \approx 1. \tag{7}$$

This equality is exact, when there is no injection, at a low Mach number and at $\mathrm{Re}^{**} \to \infty$. Condition (7) means that a correspondence is sought between two converging points in the compressible and in the incompressible stream respectively with the same Reynolds number: $\overline{\mathrm{Re}}^{**} = \mathrm{Re}^{**}$. The skin friction at an impermeable surface in a compressible stream was calculated on the basis of the Cowles hypothesis of a logarithmic segment in the velocity profile which can be described by the "wall law." Representing the test data in the form

$$\frac{u}{u_0} = f\left(\frac{u_0\rho_0 \int_0^y \rho/\rho_0 dy}{\mu_W}\right),$$

with ρ/ρ_0 determined from the Crocco integral [2], the "coefficients of skin friction in the incompressible transform stream" $\overline{C}_f$ were then found. For a compressible stream C_f was determined from the relation between the friction coefficients for a compressible and an incompressible boundary layer, according to the theory in [3] with Re → ∞:

$$\left(\frac{C_f}{C_{f_0}}\right)_{Re^{**}} = \Psi_M, \tag{8}$$

inasmuch as this case is close to the case where $\overline{\mu}/\sigma\mu_W = 1$. The time calculated values of C_f, with a maximum scatter of 17%, for an impermeable surface and for Ma = 4 were close to the values obtained by the Baronti–Libbi method (for Ma < 4 the scatter of C_f values was smaller [9]). At the same time, the $\overline{C}_f$ values agreed with the Karman formula.

The friction coefficients at a permeable surface were determined for the "incompressible transform stream" on the basis of Stevenson's law

$$\frac{2\sqrt{\frac{\overline{C}_f}{2}}}{\overline{j}_{incompr}}\left\{\left(1+\frac{\overline{j}_{incompr}}{\overline{C}_f/2}\right)^{1/2}-1\right\} = \frac{1}{K}\ln\frac{u_0\rho_0\int_0^y \frac{\rho}{\rho_0}dy\sqrt{\frac{\overline{C}_f}{2}}}{\overline{\mu}}+C, \tag{9}$$

with K = 0.41 and C = 4.9.

Equation (9) includes, besides $\overline{C}_f$, also the unknown injection factor $\overline{j}_{incompr}$ for the incompressible stream. The relation between $\overline{j}_{incompr}$ and the given injection factor $\overline{j}$ was established as

$$\overline{j}_{incompr} = \overline{j}/\Psi_M, \tag{10}$$

from the conditions that the friction is Newtonian and that both streams satisfy the momentum equations at the wall. The validity of expression (10) is indicated by the fact that, in Dorodnitsyn variables, the velocity profiles at different values of the Mach number are identical if the value of b_M is the same.

The value of C_f for each gas injection rate and for a given Mach number was found from the measured profile, with the aid of a computation grid set up according to Eqs. (9) and (10). The value of Ψ_M in (10) was determined with the aid of the relation $r = f(b_M)$ (Fig. 1). The calculations became less precise with an increasing injection factor, because the logarithmic segment in the measured profile was becoming shorter and the density of the computation grid was becoming denser. A typical graph for determining $\overline{C}_f$ is shown in Fig. 3. The scatter of $\overline{C}_f$ values here is 15% with $\overline{b} = 0.5$ and already 50% or more with $\overline{b} = 1$. From $\overline{C}_f$ we calculated C_f according to relation (8) with the recovery factor also taken into account. In Fig. 4 are shown the results of friction calculations with the injection factor varying over a wide range (b_M = 1.0-5.2) and the Mach number from 0.3 to 4. These results are also compared here with those which Dershin et al. have obtained by measuring the friction coefficient at Ma = 3.2 with a floating probe [10]. The data on friction as well as those on heat transfer can be accurately enough described by the same theoretical relations (3), (5). This correspondence (Fig. 2, 4) confirms that the modified Reynolds analogy holds true for a turbulent boundary layer with mass transfer during a supersonic flow (at least at a Mach number up to Ma = 4). This is also confirmed by the approximate similarity between the measured velocity and temperature fields in supersonic streams at various values of the injection factor.

In conclusion, we note that, despite the rather close correlation between friction coefficients found from measured velocity profiles and those calculated according to relations (3)-(5), the accuracy of the proposed method (the transformation method) of determining the magnitude of friction must still be checked out. For this, one requires a series of friction coefficient measurements with a floating probe, such as in [10]. In addition, are needed also the results of velocity field measurements in the boundary layer which, unfortunately, are not given in [10].

NOTATION

$\bar{\Psi} = \Psi / \Psi_M \Psi_{\bar{\psi}}$ is the referred heat transfer (friction) function;
St_0 is the Stanton number for standard isothermal conditions without mass transfer;
ω is the dimensionless velocity across the boundary layer;
ρ is the density;
$\bar{j} = \rho_w W_w / \rho_0 W_0$;
$b = \bar{j}/St_0$;
$b_M = \bar{j}/St_0 \Psi_M$;
$b_1 = \bar{j}/St_0 \Psi$ are the injection parameters;
r is the recovery factor;
Ma is the Mach number;
$\psi = T_w/T_0$;
$\psi^* = T_w^*/T_0$;
$\bar{\psi} = T_w/T_w^*$.

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